Polaron states of electrons in the anisotropic surface over liquid helium

2000 J. Phys.: Condens. Matter 12 7341

(http://iopscience.iop.org/0953-8984/12/33/303)

J. Phys.: Condens. Matter **12** (2000) 7341–7357. Printed in the UK PII: S0953-8984(00)14411-X

Polaron states of electrons in the anisotropic surface over liquid helium

Sviatoslav S Sokolov†‡, Antônio Carlos A Ramos† and Nelson Studart†

† Departamento de Física, Universidade Federal de São Carlos, 13565-905 São Carlos, São Paulo, Brazil

‡ B I Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, 61164 Kharkov, Ukraine

Received 1 June 2000

Abstract. The energetics and transport properties of the polaron in the anisotropic surface over liquid helium are investigated. The localization radii and the energy of the ground and excited states are calculated using the variational method within the hydrodynamic model of the polaron. In particular, we have considered maximal anisotropy which corresponds to the system of electrons in quasi-one-dimensional channels over liquid helium. The polaron binding energy is found and the temperature for the polaron formation is estimated to be below 0.1 K. Solving the hydrodynamic equations for fluid velocities, the polaron mobilities along and across the channel are determined. The possibility of experimental observation of polarons by measuring the frequency of spectroscopic transitions as well the mobility as functions of the holding electric field is addressed.

1. Introduction

An electron together with its self-induced polarization in a medium forms a quasiparticle which has been named a polaron. Besides its importance as a standard theoretical model of a fermionic particle coupled to a boson scalar field, the polaron has been observed in some physical systems. In particular, there has been great interest in the search for polaron states for surface electrons levitated over liquid helium whose properties render the system a good candidate for producing a polaron of reduced dimensionality. It is well known that electrons over helium form a plasma in which the Coulomb coupling can be varied over a wide range, from the gas regime at low densities to the triangular Wigner crystal in the opposite regime. In the gas regime and at low temperatures, electrons are scattered by surface oscillations (ripplons in their quantized form) and the electron–ripplon coupling can be considered perturbatively. The polaron state is found in the regime in which the surface electron state is still at low density, but its strong coupling to the deformation surface cannot be treated as a perturbation. This makes the nature of the polaron state much more complicated. It is not surprising that, although predicted theoretically a long time ago [1, 2], the surface polaron over liquid helium has remained the focus of a great amount of experimental work during the last two decades [3–5].

Theoretical approaches to the investigation of the surface polaron over helium are of two types. One is based on the description of the dimple state (the electron plus the deformation of the isotropic helium surface due to the pressing field) through the minimization of the total-energy functional of the dimple which leads to a system of coupled equations of motion [1, 2, 6, 7]; the transport properties are evaluated in terms of classical hydrodynamical equations since the polaron has a high effective mass [1, 2]. The other involves the concept of

0953-8984/00/337341+17$30.00

a Fröhlich-like polaron (a single electron coupled to ripplons) [8]. The conductivity has been calculated in terms of a force–force correlation function within the linear response theory [9]. Despite the great difference of the methods, the final results for the structure of the ripplonic polaron derived by the two approaches show fair qualitative and quantitative agreement [10].

In a seminal paper, Shikin and Monarkha [1] determined the ground state of the electron in an *isotropic* surface deformation through the solution of the Schrödinger equation for the electron trapped in the dimple and the mechanical equilibrium equation using a Fourier–Bessel transform, in view of the circular symmetry of the equations. The profile of the surface deformation was taken in the harmonic approximation (HA) which allows one to obtain the localization length of the electron from the Gaussian wave function, i.e. the polaron radius. Later Monarkha [2] using a variational method (VM) was able to obtain the localization length from the minimization of the polaron energy calculated with a trial Gaussian wave function. In these works, the influence of external fields and the thickness of the helium film on the static and dynamical properties of the polaron was investigated. Marques and Studart [11] have solved the Schrödinger equation and the mechanical equilibrium equation in a self-consistent way, obtaining both the electron wave function and the profile of the dimple. The comparison of the numerical results with those from the HA and the VM shows that the VM provides a more exact description of the electron wave function than the HA. More recently Farias and Peeters [12] have also used the VM for determining both ground and excited polaron states, taking into account the effect of a positive impurity charge localized in the substrate that supports the helium film.

Recently, there has been a growing interest in the effects of a corrugated helium surface on the properties of surface electrons. One motivation is the desire to make use of suspended helium films [13] to increase electron densities, which are limited in the case of bulk helium by a surface instability and by the impossibility of obtaining high-mobility electrons on a thin film, in which high densities could be achieved, due to surface roughness of the substrate. The other is the desire to confine surface electrons in one and zero dimensions—as has been realized in semiconductor heterostructures.

Quasi-one-dimensional (Q1D) electron systems on the surface of liquid helium have been realized by either geometric or electrostatic mechanisms which provide the distortion of the helium surface, and a confining electric field holds the surface electrons along the liquid channels formed. Multi-wire systems have been created using dielectric optical gratings [14], substrate wrapping by nylon threads [15], and metallic gate structures [16]. A single wire was also produced using a sharply bent polymer film [17] and metallic strips on a printed circuit board [18]. In such a Q1D system, the electron motion is restricted by, in addition to the quantum well due to the holding electric field in the direction normal to the liquid surface, a lateral confinement, and can be modelled in the first order of approximation by a harmonic potential. Under such conditions, the formation of an *asymmetric* polaron may also become possible.

In this paper, we address the question of polaron properties by using the hydrodynamic approach in the case of an anisotropic potential $U(x, y)$ due to surface corrugation and in particular for the Q1D electron system described by a parabolic confinement in the y-direction. The general formalism can be applied for both circularly symmetric and asymmetric polaron states. We consider the properties of both ground and excited polaron states and the anisotropic transport properties of the polaron are investigated for surface electrons on ^{4}He and ^{3}He [19].

The paper is organized as follows. The general formalism and the main relations are described in section 2. In section 3 we analyse the properties of the ground and excited polaron states. In section 4 we investigate the polaron mobility when a driving electric field is applied parallel to the liquid surface. In section 5 we summarize our main results.

2. Theoretical formalism

Electrons above the free surface ($z = 0$) of a helium film with thickness d are prevented from penetrating into the liquid because of a high potential barrier ($\sim$1 eV) at the liquid–vapour interface. If an electric field $E_\perp$ is applied in the z-direction, the electrons are trapped in a quantum well due to image forces coming from the liquid helium and the substrate and determined by the potential $V(z) = eE_\perp z - \Lambda_0/z - \Lambda_1/(z+d)$ where $\Lambda_0 = e^2(\epsilon_{\rm He}-1)/4(\epsilon_{\rm He}+1)$, $\Lambda_1 = e^2\epsilon_{\rm He}(\epsilon_s-\epsilon_{\rm He})/(1+\epsilon_{\rm He})^2(\epsilon_{\rm He}+\epsilon_s)$ with $\epsilon_{\rm He}$ and ϵ_s the dielectric constants of helium and the substrate respectively. If the barrier height is approximated as infinity, the condition $\Psi(x, y, z = 0) = 0$ for the electron wave function must be fulfilled for a flat surface. The situation changes drastically if we take into account the surface deformation $\xi(x, y)$. Now the boundary condition for $\Psi(x, y, z)$ has to be imposed at $z = \xi(x, y)$. Shikin and Monarkha [20] show that the transformation to a new variable $z' = z - \xi(x, y)$ allows one to avoid the perturbation in the boundary condition for $\Psi(x, y, z)$, leading, however, to modifications in the Schrödinger equation. For $\xi(0, 0) \ll d$ and $\langle z\rangle \ll d$, where $\langle z\rangle$ is the mean electron distance from the surface, these modifications result, in particular, in the dependence of $V(z)$ on an effective holding electric field given by $E_\perp^* = E_\perp + \Lambda/ed^2$ with $\Lambda = e^2(\varepsilon_s - 1)/4(\varepsilon_s + 1)$ where we take $\varepsilon_{\rm He} \simeq 1$, and the appearance of an additional term $eE_\perp^*\xi(x, y)$.

The electron motion along the plane (x, y) in the presence of a magnetic field B in the z-direction is described by the Schrödinger equation

$$\frac{1}{2m}\left[\left(\widehat{p}_x + \frac{eB}{2c}y\right)^2 + \left(\widehat{p}_y - \frac{eB}{2c}x\right)^2\right]\psi(x, y) + [eE_\perp^*\xi(x, y) + U(x, y)]\psi(x, y) = \varepsilon\psi(x, y). \tag{1}$$

Here $\widehat{p}_x$ and $\widehat{p}_y$ are the x- and y-components of the momentum operator and we have chosen the symmetric gauge of the vector potential $\vec{A} = (-By/2, Bx/2, 0)$. Note that due to the explicit (x, y) dependence of the potential term in equation (1), $\widehat{p}_x$ and $\widehat{p}_y$ are not conserved. The confinement potential $U(x, y)$ will be considered as a general anisotropic parabolic well given by

$$U(x, y) = \frac{m\omega_0^2}{2}(\alpha x^2 + y^2) \tag{2}$$

where α is the anisotropic parameter. For $\alpha = 1$, we restore the circular symmetry and $\alpha = 0$ corresponds to the case of a Q1D electron considered in references [21,22]. Then equation (1) can be rewritten as

$$-\frac{\hbar^2}{2m}\nabla^2\psi - \frac{\hbar\omega_c}{2}\widehat{L}_z\psi + \left[\frac{m}{8}(\omega_x^2x^2 + \omega_y^2y^2) + eE_\perp^*\xi\right]\psi = \varepsilon\psi \tag{3}$$

where ∇ is the 2D gradient, $\omega_x^2 = \omega_c^2 + 4\alpha\omega_0^2$ and $\omega_y^2 = \omega_c^2 + 4\omega_0^2$ with $\omega_c = eB/mc$ the cyclotron frequency, and $\widehat{L}_z$ is the angular momentum operator along z which is also not conserved for $\alpha \neq 1$ when the axial symmetry is lost. As a consequence, the second term in equation (3) does not contribute to energy eigenvalues ε if $\psi(x, y)$ is taken real. If we assume that ψ is real and vanishes at infinity, the electron energy can be calculated from

$$\varepsilon = \int\int\left[\frac{\hbar^2}{2m}(\nabla\psi)^2 + \frac{m}{8}(\omega_x^2x^2 + \omega_y^2y^2)\psi^2 + eE_\perp^*\xi(x, y)\psi^2\right]\mathrm{d}x\,\mathrm{d}y. \tag{4}$$

The total energy of the complex 'electron + dimple' can be defined as

$$W = \varepsilon + \frac{\sigma}{2}\int\int\left[(\nabla\xi)^2 + k_c^2\xi^2\right]\mathrm{d}x\,\mathrm{d}y \tag{5}$$

where $k_c^2 = \rho g'/\sigma$ is the capillary constant, $g' = g(1 + 3f/\rho g d^4)$, f is the van der Waals coupling constant for the coupling of the helium to the substrate, σ and ρ are the surface tension coefficient and the mass density of helium, respectively, and g is the gravitational acceleration. As is seen from equation (5), the inequality $W > \varepsilon$ is always satisfied. Minimizing equation (5), we obtain the mechanical equilibrium equation

$$\sigma(\nabla^2\xi - k_c^2\xi) = eE_\perp^*\psi^2. \tag{6}$$

Note that the quantity $eE_\perp^*\psi^2$ plays the role of an electron pressure on the liquid surface.

As is seen from equations (3)–(5), the asymmetry of the electron motion in the x- and y-directions for $\omega_x \neq \omega_y$ makes it inappropriate to use the Fourier–Bessel transform in polar coordinates as in previous works. Here we use the 2D Fourier transform for $\xi(x, y)$ as follows:

$$\xi(x, y) = \sum_{\boldsymbol{k}} \xi_{\boldsymbol{k}} e^{i(k_x x + k_y y)} \qquad \xi_{\boldsymbol{k}} = \frac{1}{S}\int \xi(x, y) e^{-i(k_x x + k_y y)}\, dx\, dy \tag{7}$$

and a similar transform for ψ. Here $\boldsymbol{k}$ is the 2D wave vector and S is the surface area. Using equation (7) one can easily obtain the following expression which connects the Fourier transforms of $\xi(x, y)$ and $\psi^2(x, y)$:

$$\xi_{\boldsymbol{k}} = -\frac{eE_\perp^*}{\sigma(k^2 + k_c^2)}\left[\psi^2(x, y)\right]_{\boldsymbol{k}}. \tag{8}$$

Equations (7) and (8) can be used to eliminate $\xi(x, y)$ from equations (4) and (5) and obtain the total energy of the polaron in terms only of the electron wave function and its Fourier transform as

$$W = \int\int \left[\frac{\hbar^2}{2m}(\nabla\psi)^2 + \frac{m}{8}(\omega_x^2 x^2 + \omega_y^2 y^2)\psi^2\right] dx\, dy - \frac{(eE_\perp^*)^2}{2\sigma}\sum_{\boldsymbol{k}} \frac{\left[\psi^2(x, y)\right]_{\boldsymbol{k}}\left[\psi^2(x, y)\right]_{-\boldsymbol{k}}}{k^2 + k_c^2}. \tag{9}$$

The electron energy ε has the same expression as W except for the absence of the factor 2 in the last term of equation (9). For the sake of completeness, we can rewrite the resulting Schrödinger equation after removing $\xi(x, y)$ from equation (3), as

$$-\frac{\hbar^2}{2m}\nabla^2\psi - \frac{\hbar\omega_c}{2}\widehat{L}_z\psi + \frac{m}{8}(\omega_x^2 x^2 + \omega_y^2 y^2)\psi - \frac{(eE_\perp^*)^2}{\sigma}\left(\sum_{\boldsymbol{k}} \frac{[\psi^2(x, y)]_{\boldsymbol{k}}}{k^2 + k_c^2} e^{i(k_x x + k_y y)}\right)\psi = \varepsilon\psi. \tag{10}$$

Equation (10) is quite general and can be solved self-consistently, and the results are used to evaluate the total polaron energy W and surface profile $\xi(x, y)$. Furthermore, any additional potential energy can be included in equation (10) in a straightforward way. In particular, the effects of an impurity charge [12] can also be studied using a modified version of equation (10) if the conditions $\xi(0, 0), \langle z\rangle \ll d$ are still valid. Unfortunately, equation (10) can only be solved by numerical methods and the procedure is cumbersome even in the symmetric case of $\alpha = 1$ [11]. We prefer to make reasonable guesses about the structure of $\psi(x, y)$ and obtain analytical results for the energetics and transport properties of the asymmetric polaron.

We also hereafter limit our consideration to the most studied case of the Q1D system model [21,22], where the surface electrons are confined by a lateral potential well corresponding to the completely anisotropic limit, $\alpha = 0$, i.e. $U(y) = m\omega_0^2 y^2/2$ with the characteristic frequency defined as $\omega_0 = \sqrt{eE_\perp^*/mR}$, where R ($\sim 10^{-4}$–10^{-3} cm) is the radius of curvature of the

liquid in the channel. In the absence of a magnetic field, the spectrum for the free-electron motion is given by $E(k_x, n) = \hbar^2 k_x^2/2m + \hbar\omega_0(n + 1/2)$ and the electron localization in the y-direction estimated by the parameter $L_0 = (\hbar/m\omega_0)^{1/2} \ll R$, and, for this reason, curvature effects play no significant role because $E_\perp^*$ pushes the electron to the bottom of the channel. This parameter is somewhat changed by the presence of a magnetic field [23]. Typical values of L_0 are of the order of 10^{-6} cm [21, 22] and the condition $L_0 \ll R$ is satisfied with great accuracy. Obviously, for the polaron state the localization parameter along the y-direction will be significantly modified.

3. Polaron energetics

3.1. The ground state

On the basis of the general structure of equation (10), we choose the trial function for describing the ground state of the polaron as

$$\psi_0(x, y) = \frac{1}{\pi^{1/2}(\ell_x \ell_y)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{x^2}{\ell_x^2} + \frac{y^2}{\ell_y^2}\right)\right] \tag{11}$$

where ℓ_x and ℓ_y are the electron localization lengths in the x- and y-directions, respectively. Substituting equation (11) into equations (6)–(9), one obtains the surface deformation $\xi_0(x, y)$ and the total energy W_0 of the polaron ground state as

$$\xi_0(x, y) = -\frac{eE_\perp^*}{4\pi^2\sigma} \int \mathrm{d}k_x \int \mathrm{d}k_y \, \frac{\exp[-(k_x^2\ell_x^2 + k_y^2\ell_y^2)/4]\cos(k_x x)\cos(k_y y)}{k_x^2 + k_y^2 + k_c^2} \tag{12}$$

and

$$W_0 = -\frac{(eE_\perp^*)^2}{8\pi^2\sigma} \int \mathrm{d}k_x \int \mathrm{d}k_y \, \frac{\exp[-(k_x^2\ell_x^2 + k_y^2\ell_y^2)/2]}{k_x^2 + k_y^2 + k_c^2} + \frac{\hbar^2}{4m}\left(\frac{1}{\ell_x^2} + \frac{1}{\ell_y^2}\right) + \frac{m}{16}\left[\omega_c^2\ell_x^2 + (\omega_c^2 + 4\omega_0^2)\ell_y^2\right]. \tag{13}$$

The ground-state energy $\varepsilon^{(0)}$ of the electron has the same form except for the coefficient of the integral which in this case is $(eE_\perp^*)^2/4\pi^2\sigma$. In the limit $k_c^2(\ell_x^2 + \ell_y^2) \ll 1$, the integral in equation (13) can be evaluated analytically, resulting in

$$W_0 \simeq -\frac{(eE_\perp^*)^2}{4\pi\sigma} \ln\left[\frac{2\sqrt{2}}{\sqrt{\gamma}k_c(\ell_x + \ell_y)}\right] + \frac{\hbar^2}{4m}\left(\frac{1}{\ell_x^2} + \frac{1}{\ell_y^2}\right) + \frac{m}{16}\left[\omega_c^2\ell_x^2 + (\omega_c^2 + 4\omega_0^2)\ell_y^2\right] \tag{14}$$

where $\gamma = \exp C$, and $C = 0.5772\ldots$ is the Euler–Mascheroni constant. The value of $\varepsilon^{(0)}$ is nearly the same, but with the coefficient 2 instead of 4 in the denominator of the first term of equation (14).

The localization lengths ℓ_x and ℓ_y, which appear in equations (11)–(14), have been evaluated by a few methods. In particular, a HA similar to that used in reference [1] can be applied which is based on the following approximate expression for the surface deformation:

$$\xi_0(x, y) \simeq \xi(0, 0) + \frac{1}{2}\left[\xi''_{xx}(0, 0)x^2 + \xi''_{yy}(0, 0)y^2\right]. \tag{15}$$

This expansion is similar to that of the potential energy near its minimum value in the 2D oscillatory problem and reduces the problem of the electron motion in the dimple to that of the motion in a parabolic confinement potential, by defining ℓ_x and ℓ_y as localization parameters for the 2D harmonic oscillator. We prefer however to use the VM, which allows us to obtain

ℓ_x and ℓ_y from the minimization conditions for the energy W_0, i.e. $\partial W_0/\partial \ell_x = \partial W_0/\partial \ell_y = 0$. Note that in the VM the key point for consideration is the expression for the polaron energy, given by equation (14), whereas the structure of the dimple potential is less important and is taken into account through equation (6). From the experimental point of view, using the VM seems more convenient because the characteristic value of the helium surface depression in the centre of the dimple is about 10^{-8} cm for $E_\perp^* \sim 3$ kV cm^{-1} which is impossible to detect directly. Hereafter, this estimate is based on actual holding fields in experiments on electrons along Q1D channels on bulk helium, where the effects of the film thickness are neglected [14, 17]. On the other hand, as we will see, the energy gap between the ground and excited polaron states, calculated using the VM, may in principle be accessible experimentally.

In the VM, ℓ_x and ℓ_y can be found from the roots of the system of equations

$$\frac{1}{\ell_x^4} - \frac{1}{L_F^2 \ell_x(\ell_x+\ell_y)} - \frac{1}{L_B^4} = 0 \qquad \text{and} \qquad \frac{1}{\ell_y^4} - \frac{1}{L_F^2 \ell_y(\ell_x+\ell_y)} - \frac{1}{L_0^4} - \frac{1}{L_B^4} = 0 \tag{16}$$

where $L_F^2 = 2\pi\sigma\hbar^2/m(eE_\perp^*)^2$ and $L_B^2 = 2\hbar/m\omega_c$. Equations (16) have been solved numerically. The results are presented in figure 1 as functions of the holding field for some values of the magnetic field and for ^{4}He as the liquid substrate. Analytical solution of equations (16) is possible in some limiting cases. For $B = 0$ and very high holding fields $E_\perp^* \gg 4$ kV cm^{-1} satisfying the condition $L_F \ll L_0$, one obtains $\ell_x \simeq \ell_y \simeq \sqrt{2}L_F$ which is the localization parameter for the symmetric polaron where the effects of the lateral confinement along $\boldsymbol{y}$ are negligible. For holding fields in the range $1 < E_\perp^* < 3$ kV cm^{-1}, in the opposite limit $L_F \gg L_0$, one has $\ell_x \simeq L_F$ and $\ell_y \simeq L_0$ which correspond to the numerical results shown in figure 1. Hence the localization length ℓ_y is almost the same as that for the electron moving freely along the Q1D channel. The numerical estimates at $B = 0$ are $\ell_x \sim 10^{-5}$ cm and $\ell_y \sim 10^{-6}$ cm for $1 < E_\perp^* < 3$ kV cm^{-1}. These values are significantly smaller than the radius of curvature R. This means that not only is the condition $\ell_y \ll R$, which supports the validity of our Q1D confinement model, fulfilled, but also the condition $\ell_x \ll R$ is satisfied. The application of the magnetic field leads to a decrease of the localization lengths in comparison with those for $B = 0$. For high B ($\omega_c \gg \omega_0$), one obtains from equations (16) $\ell_x \simeq \ell_y \simeq \sqrt{2}L_F$ for $L_F \ll L_B$ and $\ell_x \simeq \ell_y \simeq L_B$ for $L_F \gg L_B$. We point out that ℓ_x and ℓ_y, calculated using the HA, give different asymptotic values: $\ell_x \simeq \ell_y \simeq L_F$ for $L_F \ll L_0, L_B$, while for $B = 0$, the results are $\ell_x \simeq L_F/\sqrt{2}$ and $\ell_y \simeq L_0$ at $L_F \gg L_0$. For high magnetic fields and $L_F \gg L_B$, one has $\ell_x \simeq \ell_y \simeq L_B$. It is interesting also to note that if the VM is used for the energy of the electron trapped in the dimple ($\varepsilon^{(0)}$ instead of W_0), the results are the same as those obtained in the HA. One can conclude that the solutions of equations (16) yielding $\ell_x = \ell_y$, which is the case for a symmetric polaron, appear either in the limit of very high holding field (small L_F) or for high magnetic fields ($\omega_c \gg \omega_0$, $L_F \gg L_B$).

Using the localization lengths from equation (16), we depicted in figure 2 the polaron energy as a function of the holding field for some values of the magnetic field. For $\ell_x \simeq L_F$ and $\ell_y \simeq L_0$ and also $B = 0$, equation (14) is rewritten as

$$W_0 \simeq -\frac{(eE_\perp^*)^2}{4\pi\sigma} \ln \frac{2\sqrt{2}}{\sqrt{\gamma}k_c L_F} + \frac{\hbar^2}{4mL_F^2} + \frac{\hbar\omega_0}{2}. \tag{17}$$

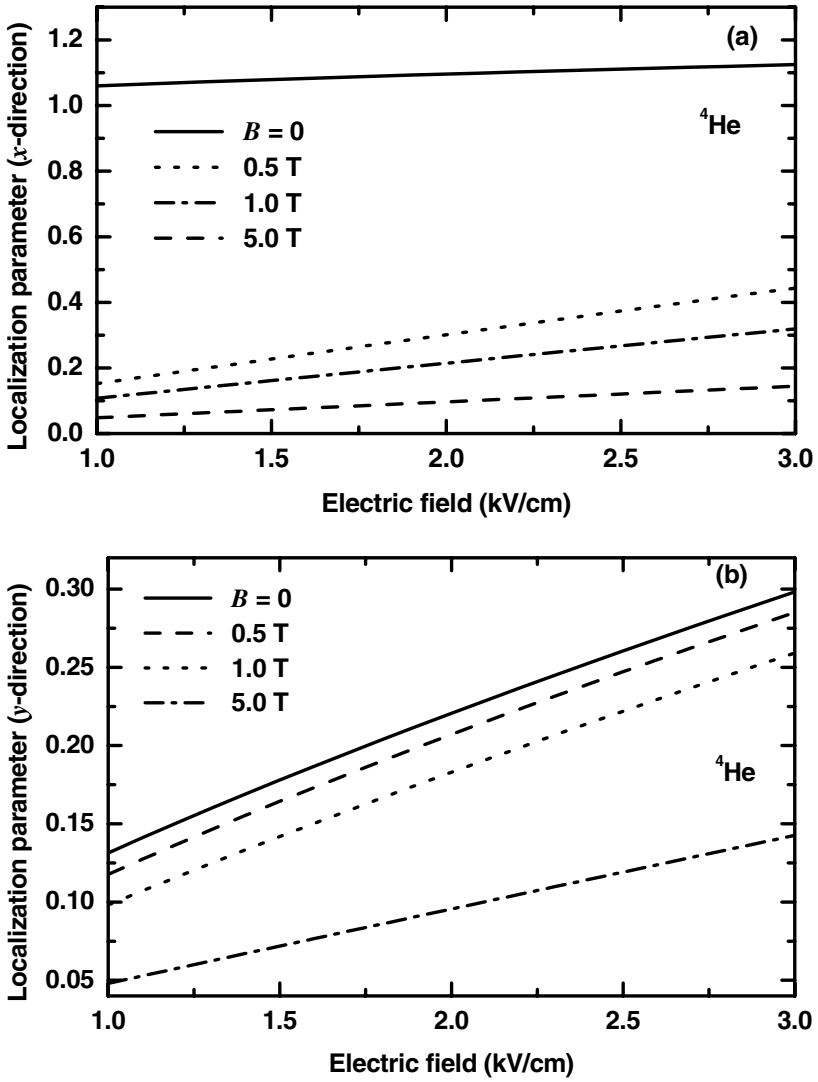

Figure 1. Localization parameters ℓ_x (a) and ℓ_y (b) in units of $L_F = (2\pi\hbar^2\sigma/m)^{1/2}/(eE_\perp^*)$ for the polaron ground state on the surface of ^{4}He as a function of the holding electric field $E_\perp^*$ for some values of the magnetic field.

Defining the binding energy E_b as the energy of the polaron state minus the electron energy $\hbar\omega_0/2$ of the free motion in the lowest subband of the lateral potential, one obtains

$$E_b \simeq -\frac{(eE_\perp^*)^2}{4\pi\sigma}\ln\frac{2\sqrt{2}}{\sqrt{\gamma}k_cL_F} + \frac{\hbar^2}{4mL_F^2}. \tag{18}$$

Note that we have considered very low temperatures $T \ll \hbar\omega_0$ where only the lowest subband $n = 0$ is occupied (for the classical surface electron system, $k_x \sim \sqrt{mT}/\hbar$). The polaron state is preferable energetically at $T < |E_b|$; otherwise thermal motion can liberate electrons from the dimple. We estimate $E_b \simeq -0.03$ K and $E_b \simeq -0.3$ K for holding electric fields of 1 V cm^{-1} and 3 kV cm^{-1}, respectively, in the case of a ^{4}He substrate. One can

conclude that the energetic conditions for the formation of the polaron in the Q1D electron system on the liquid helium surface are almost the same as in the case of the 2D symmetric polaron.

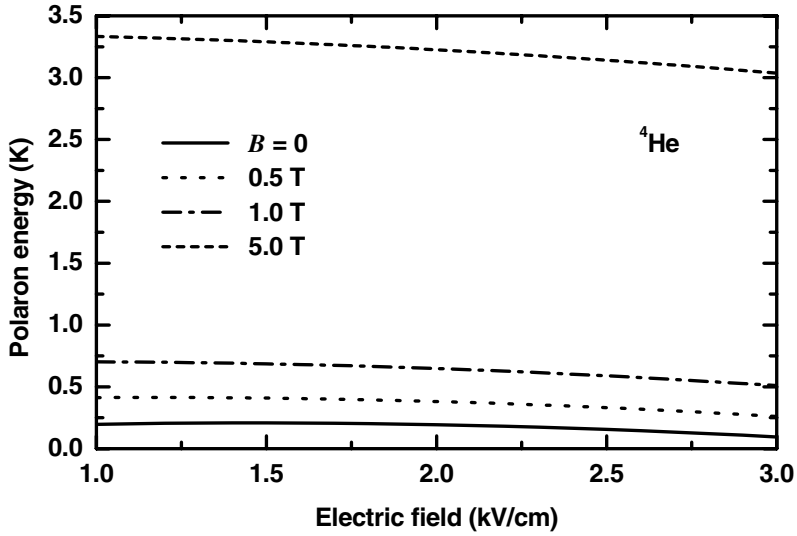

Figure 2. The total energy of the polaron ground state as a function of the holding electric field on the surface of ^{4}He.

For magnetic fields satisfying the conditions $\omega_c \gg \omega_0$, $L_F \ll L_B$, the binding energy, defined by extracting the cyclotron energy $\hbar\omega_c/2$ of the lowest Landau level from W_0 in equation (14), can be written as

$$E_b \simeq -\frac{(eE_\perp^*)^2}{4\pi\sigma} \ln \frac{\sqrt{2}}{\sqrt{\gamma} k_c L_B} \tag{19a}$$

which agrees with the asymptotic value of E_b for the symmetric polaron ($\omega_0 = 0$) for very high B. For $B = 5.5$ T ($\omega_c = 10^{12}$ s^{-1}), we estimate $L_B = 1.52 \times 10^{-6}$ cm and $E_b \simeq -0.36$ K for $E_\perp^* = 3$ kV cm^{-1}, which is very close to that for $B = 0$.

We have also estimated the profile of the surface from equation (12). For large distances such that $k_c x$ and $k_c y \gg 1$, $\xi(x, y)$ decreases exponentially as a function of the distance $r = \sqrt{(x^2 + y^2)}$ in the same manner as for the symmetric polaron [11]. The value of the dimple depth at its centre can be written as

$$\xi_0(0, 0) \simeq -\frac{eE_\perp^*}{2\pi\sigma} \ln \frac{4}{\sqrt{\gamma} k_c (\ell_x + \ell_y)}. \tag{20}$$

For $B = 0$ and $\ell_x \simeq L_F$, $\ell_y \simeq L_0$, we obtain

$$\xi_0(0, 0) \simeq -(eE_\perp^*/2\pi\sigma) \ln(4/\sqrt{\gamma} k_c L_F) \simeq -1.9 \times 10^{-8} \text{ cm}$$

for $E_\perp^* = 3$ kV cm^{-1} if the liquid substrate is ^{4}He. For high B, where $L_F \gg L_B$ and $\omega_c \gg \omega_0$, one has

$$\xi_0(0, 0) \simeq -(eE_\perp^*/2\pi\sigma) \ln(2/\sqrt{\gamma} k_c L_B) \simeq -2.2 \times 10^{-8} \text{ cm}.$$

It is interesting to recover from our results those for the symmetric polaron where $\omega_0 = 0$. In this case, the x- and y-directions are equivalent and the z-component of the angular momentum is conserved. In view of this, one can replace the operator $\widehat{L}_z$ of the angular

momentum in equations (3) and (10) by its eigenvalue l_z, which leads to the new contribution $\hbar\omega_c l_z/2$ to the electron energy ε in the case of non-zero magnetic field. We should put $l_z = 0$ for the ground state of the polaron with the lowest angular momentum. Moreover, this contribution is zero for real electron wave functions in the polaron state even for non-zero angular momentum. If $\omega_0 = 0$, the solution of equations (16) is $\ell = \ell_x = \ell_y$ with

$$1/\ell^2 = 1/4L_F^2 + \sqrt{1/16L_F^4 + 1/L_B^2}$$

and the electron wave function for the ground state is written as

$$\psi_0(x, y) = \frac{1}{\pi^{1/2}\ell} e^{-r^2/2\ell^2}$$

which leads to the same results for W_0, $\xi_0(r)$ as previously found in the case of a symmetric polaron [1, 2, 11]. In particular, the polaron energy is

$$W_0 \simeq -\frac{(eE_\perp^*)^2}{4\pi\sigma}\left[\ln\frac{1}{\sqrt{\gamma}k_c L_F} - 1\right].$$

One can easily also obtain that the second-order derivative of $\xi(r)$ at the centre of the dimple is $[\xi_{rr}''(0, 0)]_0 = eE_\perp^*/2\pi\sigma\ell^2$ which shows the existence of the minimum at $r = 0$.

3.2. Excited states

We must choose a trial excited-state wave function orthogonal to the ground-state wave function given by equation (11), which as we have seen is the same as the wave function for the 2D asymmetric harmonic oscillator in Cartesian coordinates. Hence it is natural to propose the wave function of the excited states of the 2D harmonic oscillator as the trial function for the excited polaron states:

$$\psi_{10}(x, y) = \frac{\sqrt{2}x}{\pi^{1/2}(\delta_x^3\delta_y)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{x^2}{\delta_x^2} + \frac{y^2}{\delta_y^2}\right)\right]. \tag{21}$$

Evidently, another excited state $|0, 1\rangle$ can be considered with the wave function $\psi_{01}(x, y)$. The wave function ψ_{01} has the same form as ψ_{10} as is seen by replacing x by y in equation (21). We find however that the state $|1, 0\rangle$ has smaller energy than $|0, 1\rangle$. For this reason the state $|1, 0\rangle$ should be considered the first excited polaron state and we are looking for results for this state. The results for the state $|0, 1\rangle$ can be easily obtained in a straightforward way.

Following the same procedure as for the ground state, we arrive at the following expressions for the polaron energy and the profile of the dimple in the excited state:

$$W_{10} = -\frac{(eE_\perp^*)^2}{8\pi^2\sigma}\int dk_x \int dk_y \frac{(1 - k_x^2\delta_x^2/2)^2 \exp[-(k_x^2\delta_x^2 + k_y^2\delta_y^2)/2]}{k_x^2 + k_y^2 + k_c^2} + \frac{\hbar^2}{4m}\left(\frac{3}{\delta_x^2} + \frac{1}{\delta_y^2}\right) + \frac{m}{16}\left[3\omega_c^2\delta_x^2 + (\omega_c^2 + 4\omega_0^2)\delta_y^2\right] \tag{22}$$

and

$$\xi_{10}(x, y) = -\frac{eE_\perp^*}{4\pi^2\sigma}\int dk_x \int dk_y \frac{(1 - k_x^2\delta_x^2/2)\exp[-(k_x^2\delta_x^2 + k_y^2\delta_y^2)/4]\cos(k_x x)\cos(k_y y)}{k_x^2 + k_y^2 + k_c^2}. \tag{23}$$

As before, the electron energy $\varepsilon^{(10)}$ has the coefficient $-(eE_\perp^*)^2/4\pi^2\sigma$ in the first term in equation (22).

The x-dependence of $\psi_{10}(x, y)$ contributes strongly to the decrease of the electronic pressure on the liquid surface at $x = 0$, at the centre of the dimple. According to equation (6),

the pressure is proportional to $\psi_{10}^2(x, y)$. As a consequence, in the limit $k_x^2(\delta_x^2 + \delta_y^2) \ll 1$, the second-order partial derivative $[\xi''_{xx}]_{10}(0, 0)$ calculated using equation (23) is

$$[\xi''_{xx}]_{10}(0, 0) = -[\xi''_{yy}]_{10}(0, 0) \simeq -eE_\perp^*/[\pi\sigma(\delta_x + \delta_y)^2].$$

This result is a direct consequence of the structure of the trial wave function $\psi_{10}(x, y)$ and means that the function $\xi_{10}(x, y)$ has a maximum at $x = 0$ for fixed y whereas it has a minimum at $y = 0$ at fixed x, and one cannot reach a definite conclusion as to the nature of the dimple potential as a function of the two variables x and y at $x = y = 0$. Under such conditions, the use of the HA for excited states is inappropriate, because it is based on the expansion given by equation (15) near the *minimum* of the dimple potential at $x = y = 0$. For this reason, using the VM is the only consistent way to calculate the polaron properties for the excited state.

As before, the localization lengths δ_x and δ_y can be obtained, in the VM, from the conditions for a minimum of the energy W_{10} given by equation (22). For $k_c^2(\delta_x^2 + \delta_y^2) \ll 1$, we obtain

$$W_{10} \simeq -\frac{(eE_\perp^*)^2}{4\pi\sigma}\left[\ln\frac{2\sqrt{2}}{\sqrt{\gamma}k_c(\delta_x + \delta_y)} - \frac{\delta_x(2\delta_x + 3\delta_y)}{4(\delta_x + \delta_y)^2}\right] + \frac{\hbar^2}{4m}\left(\frac{3}{\delta_x^2} + \frac{1}{\delta_y^2}\right) + \frac{m}{16}\left[3\omega_c^2\delta_x^2 + (\omega_c^2 + 4\omega_0^2)\delta_y^2\right]. \tag{24}$$

Imposing that $\partial W_{10}/\partial x = \partial W_{10}/\partial y = 0$, we arrive at the system of equations

$$\begin{aligned} &\frac{3}{\delta_x^4} - \frac{1}{L_F^2\delta_x(\delta_x + \delta_y)}\left[1 + \frac{\delta_y(\delta_x + 3\delta_y)}{4(\delta_x + \delta_y)^2}\right] - \frac{3}{L_B^4} = 0 \\ &\frac{1}{\delta_y^4} - \frac{1}{L_F^2\delta_y(\delta_x + \delta_y)}\left[1 - \frac{\delta_x(\delta_x + 3\delta_y)}{4(\delta_x + \delta_y)^2}\right] - \frac{1}{L_0^4} - \frac{1}{L_B^4} = 0. \end{aligned} \tag{25}$$

In the limit of very high holding fields where $L_F \ll L_0, L_B$, we found $\delta_x \simeq 2.12L_F$ and $\delta_y \simeq 1.73L_F$. The localization parameters obtained from the minimization of $\varepsilon^{(10)}$ are $\sqrt{2}$ times smaller. If $B = 0$, the analytical solution of equations (25) can be found in the limit $L_0 \ll L_F$: $\delta_x \simeq \sqrt{3}L_F$ and $\delta_y \simeq L_0$. Finally, for high B ($\omega_c \gg \omega_0$) one obtains $\delta_x \simeq \delta_y \simeq L_B$.

We now discuss the energy gap $\varepsilon^{(10)} - \varepsilon^{(0)}$ between the ground and excited states of the electron trapped in the dimple. First, we point out that the transitions occur between electron states in different dimple profiles given by equations (12) and (23). As a consequence, our results for the excitation frequency $\omega_{10-0} = \varepsilon^{(10)} - \varepsilon^{(0)}/\hbar$ are valid when $\omega_{10-0} \ll \omega_r$, where ω_r is the characteristic frequency of ripplons involved in the formation of the dimple. This condition is satisfied for ripplons with $\hbar\omega_r \simeq T$ for $T \lesssim 0.1$ K. Our numerical results for ω_{10-0} are presented in figure 3 as functions of the holding field for zero magnetic field and two substrates, ^{3}He and ^{4}He. In the range $1 < E_\perp^* < 3$ kV cm^{-1}, the curves can be described by the analytical expression

$$\omega_{10-0} \simeq (eE_\perp^*)^2[1 + \ln 3]/4\pi\sigma\hbar$$

which changes from 9.8×10^8 s^{-1} to 8×10^9 s^{-1} for the polaron over ^{4}He which corresponds to the electron energy increasing from 7.5×10^{-3} K to 6.2×10^{-2} K. This increase is significantly smaller than $|E_b|$ calculated from equation (18), and the electron transitions from ground to excited states do not destroy the polaron state. It is interesting to note that $W_{10} - W_0 \simeq [\varepsilon^{(10)} - \varepsilon^{(0)}]/2$ is under such conditions significantly smaller than $|E_b|$ and that also $\omega_{10-0} \ll \omega_0$, whose characteristic values vary from 5.9×10^{10} s^{-1} to 1.0×10^{11} s^{-1} when $E_\perp^*$ increases from 1 to 3 kV cm^{-1}. One should emphasize that the spectroscopic frequencies ω_{01-0} for the transition from the ground to the excited state $|0, 1\rangle$ are significantly higher than ω_{10-1} and are the same as ω_0. This means that in such a spectroscopic transition the electron

energy increases from 0.4 K to 0.7 K when $E_\perp^*$ lies in the range of 1–3 kV cm^{-1}. These energies are significantly higher than $|E_b|$ and this transition should destroy the polaron state. For this reason, only the state $|1, 0\rangle$ can be considered as an excited polaron state.

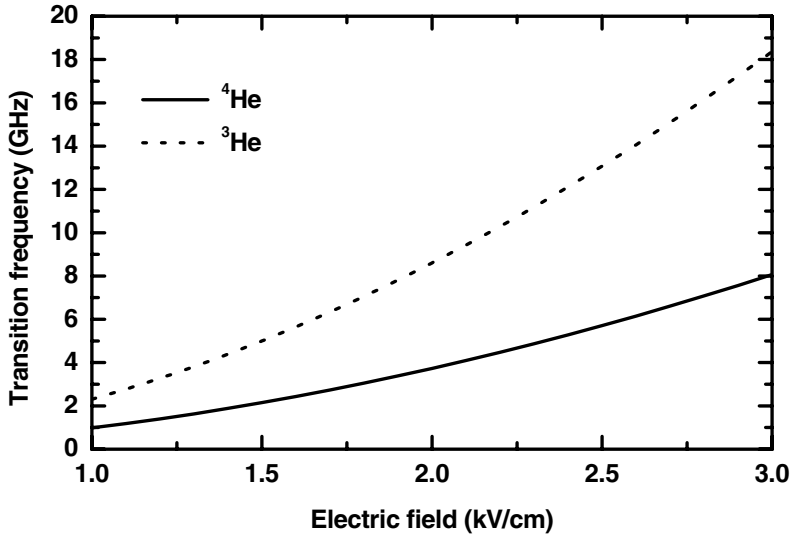

Figure 3. The frequency of the spectroscopic transitions from ground to excited electronic states in the asymmetric dimple as a function of the holding electric field, for ^{4}He and ^{3}He liquid surfaces.

We should emphasize that equation (24) allows us to estimate the transition frequency in the case of the symmetric polaron ($\omega_0 = 0$). Taking the values of δ_x and δ_y in the limit of $L_F \ll L_0$, one can easily obtain

$$\omega_{1-0} \simeq 0.65(eE_\perp^*)^2[1 + \ln 3]/4\pi\sigma$$

which is 2/3 of ω_{10-0} for the asymmetric polaron. So spectroscopic measurements can help experimentalists to give even more convincing evidence of polaron formation on helium films [4, 5].

The surface deformation for this state is given by

$$\xi_{10}(0, 0) \simeq -\frac{eE_\perp^*}{2\pi\sigma}\left[\ln\frac{4}{\sqrt{\gamma}k_c(\delta_x + \delta_y)} - \frac{\delta_x}{\delta_x + \delta_y}\right] \tag{26}$$

which follows from equation (23) in the limit of

$$k_c\sqrt{\delta_x^2 + \delta_y^2} \ll 1.$$

The absolute values of $\xi_{10}(0, 0)$ are smaller than those of $\xi_{00}(0, 0)$. For $E_\perp^* = 3$ kV cm^{-1}, $\xi_{10}(0, 0) \simeq -1.67 \times 10^{-8}$ cm for $B = 0$.

The problem of recovering the symmetric case from the previous results in the case of excited states is more complicated than for the ground state. If $\omega_0 = 0$, the excited state can be described by the trial wave function [7, 12]

$$\psi_1(r, \varphi) = \left(\frac{1}{\sqrt{\pi}\Delta^2}\right) r \exp(-r^2/2\Delta^2)\mathrm{e}^{\mathrm{i}l_z\varphi} \qquad l_z = \pm 1 \tag{27}$$

which is the eigenfunction of the angular momentum $\widehat{L}_z$ corresponding to the excited state of the 2D harmonic oscillator in polar coordinates and Δ is the single localization parameter.

However, $\psi_{10}(x, y)$ is not an eigenfunction of $\widehat{L}_z$ and hence cannot be reduced to $\psi_1(r, \varphi)$. For this reason the formal dependence of the excited polaron state on the two localization lengths δ_x and δ_y must be retained for $\omega_0 = 0$. At very large B ($L_B \ll L_F$ and $\omega_c \gg \omega_0$), we see, from equation (24), that $W_{10} = \hbar\omega_c$ and does not correspond to the energy of the first excited Landau level for electrons in the plane. In this regime, the localization scale is defined by L_B and one has $\delta_x \simeq \delta_y$. However, $\delta_x \neq \delta_y$ for small B even at very high $E_\perp^*$ ($L_F \ll L_0$) where the effects of the potential confinement are negligible and the properties of the symmetric excited polaron state should be reproduced. For $\delta_x \simeq 2.12L_F$ and $\delta_y \simeq 1.73L_F$, the symmetric polaron energy in the excited symmetric state can be written as

$$W_1 \simeq -\frac{(eE_\perp^*)^2}{4\pi\sigma}\left[\ln\frac{0.550}{k_c L_F} - 0.837\right].$$

It is interesting to know about the structure of the dimple at $r = 0$ in the limit of $\omega_0 = 0$. To make the problem clearer, we consider the second-order derivative $[\xi''_{rr}]_{10}(0)$ at the point $r = 0$, starting from the wave function given by equation (21) with $\delta_x \simeq 2.12L_F$ and $\delta_y \simeq 1.73L_F$. However, for the trial wave function $\psi_{10}(x, y)$, $[\xi''_{rr}]_{10}$ depends explicitly on φ after conversion to polar coordinates. This dependence has no physical meaning for the symmetric polaron state and results from ψ_{10} being inadequate to describe correctly the excited state in the limit of $\omega_0 = 0$. Since, in the symmetric case, the choice of the coordinate system is arbitrary, one can consider the angle-averaged value of the second-order derivative. One can easily show that $\langle[\xi'''_{rr}]_{10}(0)\rangle$ and $\langle[\xi'''_{rr}]_{10}(0)\rangle$ are almost zero in the limit of $k_c^2(\delta_x^2 + \delta_y^2) \ll 1$. Considering higher-order terms, one finds $\langle[\xi^{IV}_{rr}]_{10}(0)\rangle \simeq 3eE_\perp^*/2\pi\sigma\delta_x^3\delta_y$ which suggests a minimum at the centre of the isotropic dimple. This estimate agrees with the results of numerical calculations of reference [12]. We have observed that at large distances the function $\xi_1(r)$ decreases exponentially in the same manner as $\xi_0(r)$.

We also can extend our formalism to consider the polaron impurity states treated in reference [12]. The authors calculated the energy gap between the ground and excited electron states over a helium film considering a localization potential due to a positive impurity charge Ze located on the top of the substrate supporting the film with thickness d. It is easy to show that the impurity potential gives a correction Ze/d^2 to $E_\perp^*$ and an additional parabolic term $m\varpi^2(x^2 + y^2)/2$ with $\varpi^2 = Ze^2/md^3$, which turns the problem into one similar to that considered in the present work. Making the necessary adjustments to the expressions for $W^{(0)}$ or $\varepsilon^{(0)}$ and taking the same parameters d, k_c, and Z as in reference [12], we obtain $\varepsilon^{(10)} - \varepsilon^{(0)} \simeq 0.60$ meV which should be compared with the value 0.445 meV obtained by Farias and Peeters in a fully numerical calculation [12].

4. Polaron transport

We now investigate the transport properties of the Q1D polaron. When a driving electric field $E_\parallel$ is applied along the plane xy, the surface deformation moves together with the trapped electron inducing a field of hydrodynamic velocities in the liquid, which is accompanied by energy dissipation, and leads to a finite value of the polaron mobility [1, 2, 10, 11]. This approach is valid only in the strong-coupling limit (high $E_\perp^*$) where the self-trapped state is energetically favoured, in comparison with the weak-coupling limit where the electron is simply scattered by ripplons at $T < 1$ K [22]. In order to evaluate the polaron mobility, we employ the energy balance equation $eE_\parallel v_0 = \mathrm{d}\rho_E/\mathrm{d}t$, where v_0 is the liquid velocity at infinity, and ρ_E is the energy density dissipated. The function $\mathrm{d}\rho_E/\mathrm{d}t$ is obtained in a straightforward way by finding the normal velocity field induced by the polaron from the solution of

the Navier–Stokes equation. The calculation procedure in the asymmetric case is similar to that used in references [1, 2] for the symmetric polaron and is based on the same system of equations and boundary conditions. However, the asymmetry of the surface deformation in the x- and y-directions should be taken into account and 2D Fourier transforms like equations (7) should be employed. If the driving field is applied along the x- (y-) direction, the mobility is given by

$$\mu_{x\,(y)} = \frac{e}{2\eta S}\left[\sum_{\boldsymbol{k}} k k_{x\,(y)}^2 |\xi_{\boldsymbol{k}}|^2\right]^{-1} \tag{28}$$

where η is the helium viscosity. The summation in equation (28) can be performed analytically and the polaron mobility in the case of $E_\parallel$ along the x-direction can be written in terms of the complete elliptic integrals $\mathbf{K}(t)$ and $\mathbf{E}(t)$ as

$$\mu_x = \frac{\pi^{3/2}\sigma^2}{\sqrt{2}e(E_\perp^*)^2\eta}\left\{\frac{(\ell_x^2-\ell_y^2)\Theta(\ell_x-\ell_y)}{\ell_x\left[\mathbf{E}\left(\sqrt{1-\ell_y^2/\ell_x^2}\right)-(\ell_y/\ell_x)^2\mathbf{K}\left(\sqrt{1-\ell_y^2/\ell_x^2}\right)\right]}\right.$$
$$\left. + \frac{(\ell_y^2-\ell_x^2)\Theta(\ell_y-\ell_x)}{\ell_y\left[\mathbf{E}\left(\sqrt{1-\ell_x^2/\ell_y^2}\right)-\mathbf{K}\left(\sqrt{1-\ell_x^2/\ell_y^2}\right)\right]}\right\}. \tag{29}$$

Here $\Theta(t)$ is the step function. The general equation (29) is considerably simplified in the following limiting cases:

$$\mu_x \simeq \frac{\pi^{3/2}\sigma^2\ell_x}{\sqrt{2}e(E_\perp^*)^2\eta} \qquad \text{if } \ell_x \gg \ell_y \tag{30}$$

and

$$\mu_x \simeq \frac{\pi^{3/2}\sigma^2\ell_y}{\sqrt{2}e(E_\perp^*)^2\eta\ln(\ell_y/\ell_x)} \qquad \text{if } \ell_x \ll \ell_y. \tag{31}$$

The polaron mobility along the y-direction, μ_y, is the same as μ_x, but the index 'x' must be replaced by 'y' in equations (29)–(31) and vice versa. If $\ell_x = \ell_y = \ell$, both μ_x and μ_y reproduce the mobility of the symmetric polaron given by [1, 11]

$$\mu = \frac{\sqrt{8\pi}\sigma^2\ell}{e(E_\perp^*)^2\eta}.$$

As was seen in section 2, the limiting case $\ell_x = \ell_y = \ell \simeq \sqrt{2}L_F$ can be reached in the limit of extremely high holding fields $E_\perp^* \gg 4$ kV cm^{-1} which makes this limit experimentally inaccessible. For $1 < E_\perp^* < 3$ kV cm^{-1}, we have $\ell_x \simeq L_F \gg \ell_y \simeq L_0$. Under such conditions the longitudinal polaron mobility μ_l (along the x-direction of the channel) is given by equation (30). The transverse mobility μ_t (along the y-direction across the channel) is defined by equation (31). As follows from equations (30) and (31), $\mu_l/\mu_t \sim \ln(\ell_x/\ell_y)$ if $\ell_x \gg \ell_y$. See figure 4.

The holding-field dependences of μ_l and μ_t are depicted in figure 5. The magnetic field influences the mobility strongly because it decreases the localization parameters, as can be seen in figure 5 where polaron mobilities are plotted as functions of $E_\perp^*$ for some values of B. Since $\mu \sim \sigma^2$ and $\sigma_4 \simeq 2\sigma_3$, the mobilities for the polaron over ^{4}He are higher than those with ^{3}He as the substrate.

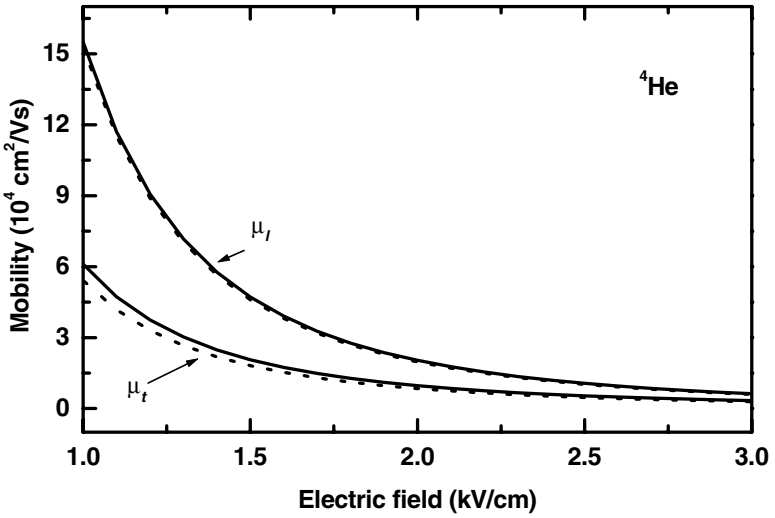

Figure 4. Longitudinal (μ_l) and transverse (μ_t) polaron mobilities over ^{4}He as functions of the holding electric field calculated by using equation (29) (solid line) and the approximate expressions (equations (30) and (31)) (dashed line) for zero magnetic field.

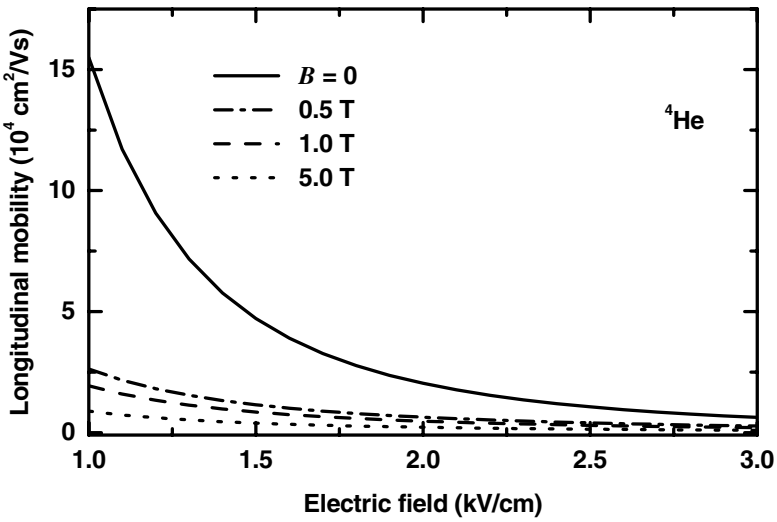

Figure 5. The longitudinal polaron mobility versus the holding electric field for some values of the magnetic field.

5. Concluding remarks

In this work, we have discussed the possibility of the formation of a polaron in the context of the anisotropic liquid helium surface. In particular, we have evaluated the energetics and transport properties of the polaron in a Q1D channel. We have considered the properties of both ground and excited states of the asymmetric polaron. The localization lengths and the ground- and excited-state energies have been determined as functions of the holding electric field within the hydrodynamical model and using a variational approach. We have obtained the

frequency of spectroscopic transitions between the ground and first levels in the asymmetric polaron states. The transport properties of the polaron along the corrugated helium surface have also been studied.

We think that the measurement of the frequency of the spectroscopic transition ω_{10-0} between the ground and excited states (see figure 4) should be useful in experimental attempts to observe the polaron state in a Q1D channel over a liquid helium surface. Indeed, our estimates indicate that ω_{10-0} differs significantly from the frequency ω_0 of spectroscopic transitions between subbands of the free-electron state in a Q1D channel, which allows one to separate the experimental signals for these two different types of spectroscopic transition.

Another possibility is measuring the mobility directly as in reference [4]. Our results indicate that the mobility of the Q1D polaron is 40% smaller than the mobility of the 2D polaron for the same holding field in the range $E_\perp^* < 3$ kV cm^{-1}. Our value of the Q1D polaron mobility, $\sim 10^4$ cm^2 V^{-1} s^{-1} for $T = 0.1$ K and $E_\perp^* = 3$ kV cm^{-1}, is three orders of magnitude smaller than the mobility of the surface electron over a flat helium surface and that for a surface electron moving freely along the channel [22], which allow us to distinguish them easily. Note that we can explore the asymmetry of the polaron mobility along and across the channel in experimental attempts to observe it.

While the above estimates show that our proposed mechanism for the polaron formation in Q1D systems on helium should be compatible with accessible experiments, we add other suggestions for corroborating or refuting our predictions. Of particular interest is the possibility of using liquid ^{3}He instead ^{4}He as the substrate for the confined Q1D electron system [19]. Recall that for ^{3}He, whose surface tension coefficient is less than half that of ^{4}He, the binding energy $|E_b|$ can be substantially larger. Also the spectroscopic transition frequencies for ^{3}He are double those for ^{4}He. Furthermore, the description of the polaron in terms of a hydrodynamic viscous model may be suspect for pure ^{4}He for temperatures below 1 K. In such a regime, the polaron mobility should be described in a more general treatment which takes into account the polaron interaction with the ballistic surface and bulk quasiparticles of superfluid helium. However, we must emphasize that experimental results [4] have been successfully interpreted by theoretical calculations based on the hydrodynamic approach for $T < 1$ K, even though this interpretation has been seriously questioned [5]. Furthermore, the hydrodynamic approach based on viscous phenomena is very satisfactory in the case of ^{3}He for T down to ~ 0.1 K. Another substrate expected to be of interest is a low-T mixture of ^{3}He and ^{4}He [25]. Recent experiments on Q1D channels have provided a higher effective holding field for electrons on helium films which could greatly enhance the absolute value of the binding energy E_b [26].

The electron states have been described in the present work in the one-electron approximation. The validity of the results in the case of finite electron densities is limited by the condition that the scale of the electron localization must be small in comparison with the mean interelectronic distance $a \simeq n_l^{-1}$ along the channel axis, where n_l is the linear electron density. For $B = 0$ and $E_\perp < 3$ kV cm^{-1} where $\ell_x \simeq L_F \gg \ell_y \simeq L_0$, the characteristic values of ℓ_x are $\sim 10^{-5}$ cm under the condition $n_l \ll 10^5$ cm^{-1}. At higher electron densities, correlation effects could strongly influence the polaron properties, and the applicability of our theory developed for the low-density limit becomes doubtful. For low electron densities, experimental difficulties may be encountered in making the measurements of the experimental signal of the response of the electron system at a low level. This signal is proportional to the electron density, which is especially restricted if we have a single Q1D channel. One possibility is that of using N_{ch} weakly coupled multi-channels [14, 16], like in multiple quantum wells in semiconductor heterostructures. If the average distance between the channels $b \gg a$, one can disregard the correlation between electrons in different channels, and the total response should

be enhanced by the factor N_{ch}. This makes the conditions more appropriate for experimental investigation of Q1D electron systems over liquid helium in the low-density limit.

We hope that the asymmetric polaron can be detected at low temperatures, probably at $T < 0.1$ K. Modern experimental methods for achieving low and ultralow temperatures would then offer the possibility of observing the polaron in the Q1D channel on the liquid helium surface.

Acknowledgments

This work was partially sponsored by the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and the Conselho Nacional de Desenvolvimento Cientifico e Tecnológico (CNPq), Brazil.

References

[1] Shikin V B and Monarkha Yu P 1973 *Zh. Eksp. Teor. Fiz.* **65** 751 (Engl. Transl. 1973 *Sov. Phys.–JETP* **38** 373)
[2] Monarkha Yu P 1975 *Fiz. Nizk. Temp.* **1** 526 (Engl. Transl. 1975 *Sov. J. Low Temp. Phys.* **1** 258)
[3] Andrei E 1984 *Phys. Rev. Lett.* **52** 1449
Mende F F, Kovdrya Yu Z and Nikolaenko V A 1985 *Fiz. Nizk. Temp.* **11** 646 (Engl. Transl. 1985 *Sov. J. Low Temp. Phys.* **11** 355)
Hu X L, Carmi Y and Dahm A J 1992 *J. Low Temp. Phys.* **89** 625
[4] Tress O, Monarkha Yu P, Penning F C, Bluyssen H and Wyder P 1996 *Phys. Rev. Lett.* **77** 2511
Tress O, Penning F C, Mistura G, Teske E and Wyder P 1998 *Physica* B **249–251** 644
[5] Rubin N A and Dahm A J 1999 *Phys. Rev. Lett.* **82** 2004
Tress O, Monarkha Yu P, Penning F C, Bluyssen H and Wyder P 1999 *Phys. Rev. Lett.* **82** 2005
[6] Sander L M 1975 *Phys. Rev.* B **11** 4350
[7] Cheng A and Platzman P M 1978 *Solid State Commun.* **25** 813
[8] Jackson S and Platzman P M 1981 *Phys. Rev.* B **24** 499
Hipólito O, Farias G A and Studart N 1982 *Surf. Sci.* **113** 394
Degani M H and Hipólito O 1985 *Phys. Rev.* B **32** 3300
[9] Saitoh M 1983 *J. Phys. C: Solid State Phys.* **16** 6995
Peeters F M and Jackson S A 1985 *Phys. Rev.* B **31** 7098
[10] For a review, see Studart N and Sokolov S S 1997 *Two-Dimensional Electron Systems in Helium and Other Substrates* ed E Y Andrei (Dordrecht: Kluwer)
[11] Marques G E and Studart N 1989 *Phys. Rev.* B **39** 4133
[12] Farias G A and Peeters F M 1997 *Phys. Rev.* B **55** 3763
[13] Volodin A P and Edel'man V S 1983 *Pis. Zh. Eksp. Teor. Fiz.* **37** 8 (Engl. Transl. 1983 *JETP Lett.* **37** 7)
Marty D 1986 *J. Phys. C: Solid State Phys.* **19** 6097
Valkering A M C, Yayama H and van der Heijden R W 1998 *J. Low Temp. Phys.* **113** 873
Valkering A M C, Klier J, Teske E, van der Heijden R W and Leiderer P 1998 *J. Low Temp. Phys.* **113** 1115
[14] Kovdrya Yu Z and Nikolaenko V A 1992 *Fiz. Nizk. Temp.* **18** 1278 (Engl. Transl. 1992 *Sov. J. Low Temp. Phys.* **18** 894)
Nikolaenko V A, Kovdrya Yu Z, Yayama H and Tomokiyo A 1997 *Low Temp. Phys.* **23** 482
Kovdrya Yu Z, Nikolaenko V N, Yayama H, Tomokiyo A, Kirichek O I and Berkutov I B 1998 *J. Low Temp. Phys.* **110** 191
[15] Kovdrya Yu Z, Nikolaenko V A, Gladchenko S P and Sokolov S S 1998 *Fiz. Nizk. Temp.* **24** 1113 (Engl. Transl. 1998 *Low Temp. Phys.* **24** 837)
[16] van Haren R J F, Acres G, Fozooni P, Kristensen A, Lea M J, Richardson P J, Valkering A M C and van der Heijden R W 1998 *Physica* B **249–251** 656
[17] Kirichek O I, Monarkha Yu P, Kovdrya Yu Z and Grigor'ev V N 1993 *Fiz. Nizk. Temp.* **19** 458 (Engl. Transl. 1993 *Low Temp. Phys.* **19** 323)
[18] Valkering A M C, Sommerfeld P K H, Richardson P J, van der Heijden R W and de Waele A T A M 1996 *Czech. J. Phys.* **46** 321
Valkering A M C and van der Heijden R W 1998 *Physica* B **249–251** 652

[19] Shirarama K, Itoh S, Suto H and Kono K 1995 *J. Low Temp. Phys.* **101** 439
Sokolov S S, Rino J P and Studart N 1997 *Phys. Rev.* B **55** 14 473
[20] Shikin V B and Monarkha Yu P 1974 *J. Low Temp. Phys.* **16** 193
[21] Kovdrya Yu Z and Monarkha Yu P 1986 *Fiz. Nizk. Temp.* **12** 1011 (Engl. Transl. 1986 *Sov. J. Low Temp. Phys.* **12** 571)
[22] Sokolov S S, Hai G-Q and Studart N 1995 *Phys. Rev.* B **51** 5977
[23] Sokolov S S and Studart N 1995 *Phys. Rev.* B **51** 2640
[24] Shikin V B and Monarkha Yu P 1975 *Fiz. Nizk. Temp.* **1** 957 (Engl. Transl. 1975 *Sov. J. Low Temp. Phys.* **1** 459)
[25] See Hallock R B 1998 *Phys. Today* 51(6) 30 and references therein
[26] Yayama H 1999 private communication